

20.2-5 PERMISSIBLE INDICES FOR MAXIMAL ISOMORPHIC SUBGROUPS OF 2-DIMENSIONAL SPACE GROUPS. Y. Billiet, Faculté des Sciences et Techniques, Université de Bretagne Occidentale, 6, avenue Le Gorgeu, 29283 Brest, France, et Recherches en Symétrie Cristallo-graphique à Sétif, A5, 35, Cité du 8 mai 1945, Sétif, Algérie.

In dimensions 2 and 3, maximal "translationengleich" subgroups and maximal non-isomorphic "klassengleich" subgroups always are finite in number. On the contrary, there exist maximal isomorphic subgroups always in infinite number for any 2- or 3-dimensional space group. However the permissible values for the index are limited. As an example, here are given the suitable values for 2-dimensional maximal isomorphic subgroups.

p1, p2, pm, pg, p2mm, p2mg: any prime integer.
om, p2gg, c2mm: any odd prime integer.

p4: 1/ any prime integer of the type $k_1^2 + k_2^2 \neq 4k_3 + 1$;

2/ any integer of the type $k_1^2 + k_2^2 = (4k_3 + 3)^2$ where $4k_3 + 3$ is a prime integer.

p4mm: 1/ the number 2; 2/ any integer of the type k^2 where k is a prime integer.

p4gm: any integer of the type $(2k+1)^2$ where $2k+1$ is a prime integer.

p3, p6: 1/ any prime integer of the type $k_1^2 + k_2^2 - k_1 k_2$;

2/ any integer of the type $k_1^2 + k_2^2 - k_1 k_2 = (3k_3 + 2)^2$ where $3k_3 + 2$ is a prime integer.

p3m1, p3lm: any integer of the type k^2 where k is a prime integer.

p6mm: 1/ the number 3; 2/ any integer of the type k^2 where k is a prime integer.

20.2-6 ON SUBGROUPS OF SPACE GROUPS.

P. Engel, Laboratory for crystallography, Freiestrasse 3, CH-3012 Berne, Switzerland.

Group-subgroup relations are of interest in several fields in crystallography. Tables of maximal subgroups of space groups are contained in the International Tables, Vol. A (D. Reidel publishing company, Dordrecht, 1983). Using these Tables however, a systematic determination of all subgroups of a space group proves to be very tedious. A novel computer program has been developed which allows a systematic determination of all subgroups of the space groups. For a space group G with given arithmetic crystal subclass and given sublattice the program calculates all possible subgroups H following an algorithm proposed by Senechal (Acta Cryst. A36 (1980) 845-850). Subgroups which are conjugate under G or alternatively under the Euclidean normalizer $N_E(G)$ are eliminated. For each Bravais class the transformation laws for the generation of all possible sublattices are derived. The space-group type of a subgroup is determined using a procedure which takes into account only geometric properties of the subgroup which remain invariant under affine transformations.

The subgroups of a space group G are classified according to their space-group type. Subgroups belonging to the same space-group type are arranged in series, each serie containing an infinite number of subgroups. It will be shown that for any space group only a finite number of series of subgroups exists. Examples will be presented.

Among the subgroups the normal subgroups are of special interest. Every normal subgroup determines the kernel of a possible homomorphism. The normal subgroups for all 2- and 3-dimensional space-group types have been determined.

Example: The normal subgroups of space-group type No. 92 $P4_1 2_1 2$ (equivalence under $N_E(G)$; $d \geq 0$; $a, c \geq 1$; the index is given in brackets).

sublattice	normal subgroups
1,0,0/0,1,0/0,0,1	$P2_1 2_1 2_1 (2)$, $C222_1 (2)$
1,0,0/0,1,0/0,0,2d+1	$P2_1 (8d+4)$
1,0,0/0,1,0/0,0,4d+1	$P4_1 (8d+2)$
1,0,0/0,1,0/0,0,4d+3	$P4_3 (8d+6)$
1,1,0/-1,1,0/0,0,1	$P222_1 (4)$, $P2_1 2_1 2_1 (4)$
1,1,0/-1,1,0/0,0,2d+1	$2 \cdot P2_1 (16d+8)$
a,0,0/0,a,0/0,0,c	$P1 (8a^2 c)$
2a,0,0/0,2a,0/a,a,c	$P1 (32a^2 c)$
a,a,0/-a,a,0/0,0,c	$P1 (16a^2 c)$
a,a,0/-a,a,0/0,a,c	$P1 (16a^2 c)$
a,0,0/0,a,0	$p1(\infty)$
a,a,0/-a,a,0	$p1(\infty)$
0,0,c	$\bar{p}1(\infty)$

20.2-7 PRESENTATION OF CRYSTALLOGRAPHIC

GROUPS BY FUNDAMENTAL POLYHEDRA. By E. Molnár, Eötvös Loránd University, Budapest, Hungary.

There is a Poincaré's method to present a discrete isometry group G by means of a fundamental polyhedron F endowed with a face identification. The identifying isometries generate the group G . The cycle relations, belonging to the edge equivalence classes of F , together with the eventual reflection relations give us the mentioned presentation of G . The author's intention, to give a so-called minimal geometric presentation for each space group, has been realized in most cases, sometimes only by concave topological polyhedra (Molnár, Beiträge zur Algebra und Geometrie (1983) 14, 33). We shall determine these polyhedra presenting minimally those 38 space groups which have semi-direct decomposition $G = G_1 \circ C_2$, where C_2 is an invariant Coxeter subgroup generated by plane reflections and G_1 is a so-called rod group leaving a straight line invariant (e.g. Koch and Fischer, Z.Kristallogr. (1978) 147, 21).

As a typical example let us consider the space group $G = R3m$. Then $G_1 = P3_1(11)$, $C_2 \cong p3m1$,