20.1-1 DIVISION OF SPACE INTO POLYHEDRA WITH ALL VERTICES SYMMETRICALLY RELATED. GEOMETRICAL AND COLORED SYMMETRY CONSIDERED. By David Harker, Medical Foundation of Buffalo, Inc., 73 High St., Buffalo, N.Y. 14203 U.S.A.

Consider an infinite three-dimensional (3D) aggregate of points in space. If each point is related to every other point by a geometrical symmetry operation (rotation about an axis, reflection through a plane, inversion through a point, translation along a certain direction, or combinations of these) the result is a 3D periodic distribution of points, each related to all the other such points in the same way (geometrically). Now connect each point to neighboring points by straight lines (edges) in such a way that every point (vertex) can be reached from every other point by moving along a sequence of edges from vertex to vertex, and such that the array of edges meeting at each point is related to the array meeting at every other point by a symmetry operation of the group which relates the points to one another. Then construct planes through a chosen point (vertex) bounded by the edges described in the last sentence, thus creating polygons with vertices at the original points and sides consisting of the edges just mentioned; this divides space into polyhedral cells. The array of these cells is such that every point in 3D space is either within a cell, on a face between two contiguous cells, in an edge, or at a vertex. Furthermore, the periodicity of the array of cells must be the same as that of the original points.

An example is the array of regular octahedra and tetrahedra defined by the nodel points of a face-centered cubic lattice. Six octahedra and eight tetrahedra have a common vertex at each lattice point.

The general and special positions of every Fedorov space group are examples of appropriate point sets. Several cell structures correspond to each such set.

If each unit cell of the periodic cell structure contains C cells, E edges, F faces, and V vertices, then

$$C + E = F + V \quad (1)$$

(H.A. Hauptman, private communication (1986)). Proper account must be taken of cells, faces, edges, and vertices shared between contiguous unit cells when using equation (1).

If each vertex is given a "color" then the phenomena of "colored symmetry" can occur in these structures. Examples will be discussed and representations will be displayed.

20.1-2 SYMMETRY CONDITIONS FOR 3-PERIODIC MINIMAL BALANCE SURFACES. By E. Koch and W. Fischer, Institute of Mineralogy, University of Marburg, FRG.

A minimal surface is defined as a surface with zero mean curvature at each of its points. It may be 3-periodic (cf. e.g. Hyde and Andersson, Z. Kristallogr., 1984, 168, 221-254). A connected intersection-free surface will be called a balance surface, if it is 3-periodic and divides the 3-dimensional space into two congruent regions such that each region is connected but not simply connected. Special examples for such balance surfaces are certain POPSs (cf. von Schnering, Z. Kristallogr., 1986, 174, 182-184). A balance surface with zero mean curvature is named a minimal balance surface.

The symmetry of any balance surface is characterized by a space-group pair G>H (index 2): the symmetry operations of H map each side of the surface onto itself, whereas the further symmetry operations of G interchange the two sides. As a consequence, all fixed points of the latter symmetry operations must lie within the balance surface and only certain space-group pairs are compatible with balance surfaces. Symmetry elements that may be contained in balance surfaces are 2-fold axes, inversion centers $\bar{1}$ and roto-inversion points $\bar{3}$ or $\bar{4}$.

If G contains additional 2-fold axes, one may distinguish different cases with respect to the connectivity of the graph formed by these axes. If such a graph is connected, in most cases it is possible to derive one (or two) related minimal balance surface(s) by simply spanning a suitable skew circuit of the graph and using the 2-fold rotations for continuation. If such a minimal balance surface exists, the graph is called a generating linear net and the circuit a generating circuit of the surface. In this way 17 types of minimal balance surfaces have been derived. The corresponding inherent symmetries are: $Im\bar{3}m$-$Pm\bar{3}m$ (2 types), $Pn\bar{3}m$-$Fd\bar{3}m$(2), $Ia\bar{3}d$-$I\bar{4}3d$(1), $Ia\bar{3}d$-$Ia\bar{3}$(1), $I4_132$-$P4_332$(2), $P6_222$-$P3_112$(1), $P6_222$-$P6_122$(1), $P4_2/nnm$-$I4_1/amd$(2), $P4_2/mcm$-$P4_2/mmc$(1), $P4_222$-$P4_122$(1), Pnnn-Fddd(1), Cmma-Imma(1), Pccm-Cccm(1).

Twofold axes forming only 2-dimensional parallel nets give rise to minimal balance surfaces like the H surface and the R surfaces (Schoen, NASA Technical Note No. D-5541, 1970). Corresponding new such types have also been derived.

In addition minimal balance surfaces exist which are spanned by non-intersecting twofold axes and/or (roto-) inversion points (e.g. $Ia\bar{3}$-$Pa\bar{3}$ and the gyroid surface with $Ia\bar{3}d$-$I4_132$).