[MS36-04] Symmetrical structures of non-intersecting cylinders Moreton Moore

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Fibre-reinforced materials usually contain threads running in one or two directions embedded in a matrix. Here we consider the possibilities of several directions in space for the fibres, considered as identical infinitely-long non-intersecting right-circular cylinders. (Cylinders which intersect, with their axes passing through a single point, with cubic crystal symmetry have already been studied [1,2].) The crystal chemists, O'Keeffe and Andersson , have considered cylinder packing in the context of crystal structures [3,4]. A comprehensive study of regular packing of fibres in three dimensions has been made by Parkhouse and Kelly [5]. Composite materials with fibres running parallel to the three <100> directions rely on the strength of the matrix material for shear strength. They will be strong in the fibre directions but relatively weak in other directions. (The packing density of the structure is $3\pi/16 \sim 0.589$.) Even employing the four <111> directions for the fibres leaves the material vulnerable to shear forces. This is because two of the <111> directions lie in one of the {110} planes and the other two <111> directions lie in a different {110} plane, which makes an angle of 90° with it. Under shear stress, this angle can change without any restoring force being provided by the fibres.
(There are two such structures with packing densities $\sqrt{3}\,\pi/8 \sim 0.680$ and $\sqrt{3}\,\pi/18 \sim 0.302$.)
Interest therefore has focussed on cylindrical structures with more than four directions, such as the six <110> directions. Moore made a model of one such structure [6] but was disappointed in that it had only tetragonal crystal symmetry: not cubic. Now it would seem that there is no <110> cylinder structure with cubic crystal symmetry; since three of the <110> directions lie in a (111) plane and to avoid intersection, displacements need to be made with components parallel to the [111] direction. To achieve cubic symmetry, triad axes are needed along the <111> directions. A structure can be made with a three-fold screw axis along one <111> direction, but not along all four. Teshima et al. [7] have reported three types of <110> cylinder packings with packing densities 0.494, 0.247 and 0.376. Teshima and Matsumoto [8] have studied the space group of last one. They have also proved that there is no <110> cylinder packing with cubic crystal symmetry.

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