Beyond Golay-Rudin-Shapiro

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I briefly recapitulate the necessary background about the original pseudorandom Golay-Rudin-Shapiro sequence (GRS) and its known generalizations [1-5]. The standard method to make the one-sided GRS based on a two-letter alphabet $\mathcal{A}_2 = \{a, b\}$ two-sided is by constructing a proto-GRS structure based on a four-letter alphabet $\mathcal{A}_4 = \{a, b, c, d\}$ and then reduce it to $\mathcal{A}_2$. In order to generalize to higher dimensions one proceeds analogically. Here I extend GRS to eight symbols (alias letters, digits or colors). I also refine the terminology introducing the designation dD GRSn for a GRS structure based on n symbols and supported by $\mathbb{Z}^d$.

The most natural support for 3D GRS8, that is a structure is based on $\mathcal{A}_8 = \{0, 1, 2, 3, 4, 5, 6, 7\}$ is $\mathbb{Z}^3$. The respective substitution is

$$
\begin{array}{llll}
0 \to \begin{array}{ccc} & 5 & 7 \\ & 2 & 3 \\ 4 & 6 & \\ 0 & 2 & \end{array} &
1 \to \begin{array}{ccc} & 5 & 4 \\ & 1 & 0 \\ 7 & 6 & \\ 3 & 2 & \end{array} &
2 \to \begin{array}{ccc} & 6 & 7 \\ & 2 & 3 \\ 4 & 5 & \\ 0 & 1 & \end{array} &
3 \to \begin{array}{ccc} & 6 & 4 \\ & 2 & 0 \\ 7 & 5 & \\ 3 & 1 & \end{array} \\
4 \to \begin{array}{ccc} & 3 & 1 \\ & 7 & 5 \\ 2 & 0 & \\ 6 & 4 & \end{array} &
5 \to \begin{array}{ccc} & 0 & 1 \\ & 4 & 5 \\ 2 & 3 & \\ 6 & 7 & \end{array} &
6 \to \begin{array}{ccc} & 3 & 2 \\ & 7 & 6 \\ 1 & 0 & \\ 5 & 4 & \end{array} &
7 \to \begin{array}{ccc} & 0 & 2 \\ & 4 & 6 \\ 1 & 3 & \\ 5 & 7 & \end{array}
\end{array}
\tag{1}
$$

The bottom matrix refers to 2D GRS8. The bottom line, in turn, refers to 1D GRS4, while the alphabet $\mathcal{A}_8$ splits into two disjoint $\mathcal{A}_4$'s. Thus 1D necessitates special treatment. As in the case of GRS4, the substitution must be applied twice.

Fig.1 shows an isometric projection of the hull of the second generation of 3D GRS8.

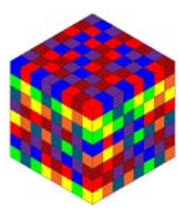

Figure 1. Aspect of hull of 3D GRS8 generation 2.

The Fourier spectrum of all GRS structures is absolutely continuous [7, 8].

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