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An assessment of approximating aspheres with more easily manufactured surfaces

Malcolm R. Howells,[a] Joel Anspach[b]* and John Bender[b]

[a]*Advanced Light Source, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA, and* [b]*Rocketdyne Division of Boeing North American, 2511C Broadbent Parkway NE, Albuquerque, NM 87107, USA. E-mail: joel.e.anspach@boeing.com*

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In designing optical systems for synchrotron radiation, one is often led to conclude that optimal performance can be obtained from optical surfaces described by conic sections of revolution, usually paraboloids and ellipsoids. The resulting design can lead to prescriptions for three-dimensional optical surfaces that are difficult to fabricate accurately. Under some circumstances satisfactory system performance can be achieved through the use of more easily manufactured surfaces such as cylinders, cones, bent cones, toroids and elliptical cylinders. These surfaces often have the additional benefits of scalability to large aperture, lower surface roughness and improved surface figure accuracy. In this paper we explore some of the conditions under which these more easily manufactured surfaces can be utilized without sacrificing performance.

Keywords: grazing-incidence mirrors; aspheres; approximation; optical fabrication.

1. Introduction

It often happens that optical designers must choose between a desired optical-surface shape, which is difficult to make, and an alternative shape which is easier to make but only approximates the desired one. Which will give the best result? If the ideal optic is not sufficiently well made, its mathematical advantages may not be realized. On the other hand, approximations have intrinsic errors which will only be small enough for certain ranges of the parameters describing the system.

In this paper we focus on grazing incidence beamline optics and study the question of approximating difficult optical surfaces with easier ones. First we consider which optical surfaces really are easier to manufacture and why. This includes manufacture by elastic bending which, under favorable conditions, can now deliver microradian accuracy. We then calculate the slope errors involved in approximating an elliptical cylinder with circular and cubic cylinder approximations. This calculation is two dimensional but it is revealing for designing condenser mirrors that deliver light into a monochromator entrance slit and for microprobe schemes based on the Kirkpatrick–Baez (KB) geometry. It shows, in particular, the very large advantage in light-gathering power of a KB microprobe with correctly designed elliptical cylinders compared with the same scheme with circular or cubic approximations. We then consider the question of approximating an ellipsoid of revolution (including a paraboloid as a special case) by a toroid. This is necessarily a three-dimensional calculation but the two-dimensional (elliptical cylinder) calculation is valid for rays in the symmetry plane of the system, so that provides a useful starting point for the calculation. Finally we summarize our conclusions. The main conclusion is the obvious one, that optical systems will only achieve their goals if they are designed from the beginning around optics that can really be manufactured within all prevailing constraints and specifications. This implies that conversations with optical fabricators early in the project are prudent so as to avoid becoming locked-in to optical-component designs that are not state of the art.

2. Which are the easiest and hardest optical-surface shapes to manufacture?

There are a number of factors which may make a mirror difficult to manufacture but here we concentrate specifically on the optical working of grazing incidence surfaces. In particular we want to make a comparison purely of surface shapes, all other things being assumed to be equal. The meaning of ‘easy’ and ‘difficult’ in this context is that easier surfaces allow better figure and finish with less effort and cost. The first step is to conclude that flats are easiest. One may object that, on a planetary lap (continuous polisher) and some other machines, long-radius spheres can be made just as easily as flats. However, this is only true when a lap of the appropriate radius is in hand. Although producing such a lap is now a routine operation, even for one-metre-class mirrors, it still requires some time and effort (cost). Further advantages of flats are that they are easy to measure and can be made with a lap which is large enough to touch the optical surface over 100% of its area all of the time.

After flats the next easiest group is certainly spheres and after that circular cylinders. These three groups have the special property that they have enough different types of symmetry operations that the normal pseudo-random motion of a polishing lap relative to the workpiece can take place while still maintaining 100% contact. This property is very advantageous in achieving a good figure and finish at the same time.

An ellipsoid of revolution, on the other hand, has only one family of symmetry operations (rotations about the axis) so the normal type of polishing motions cannot be made without loss of contact between the lap and the workpiece. Therefore, in order to polish an ellipsoid of revolution one must use a lap or other material-removal device which operates over only a small subarea of the optical surface and is small enough and flexible enough to approximate the correct surface locally. This is known as zone polishing and has been developed to a high degree of sophistication and computer control for making grazing incidence Wolter and similar optics for X-ray telescope projects. However, it is intrinsically more difficult and expensive than methods based on 100% contact and tends to leave surface errors on the spatial scale of the size of the small lap. This is the explanation why ellipsoids of revolution are more ‘difficult’ than flats, spheres and circular cylinders.

The optics that can be made without recourse to zone polishing are therefore flats, spheres and circular cylinders, and surfaces that can be made from those by elastic bending. In the simplest type of bending, a flat mirror of constant cross-sectional area is elastically bent by end couples into a tangential cylinder. This provides the quadratic (equal couples) or cubic (unequal couples) approximation to an ellipse. A more general type of bending of an initially flat mirror involves manipulation of the cross section,

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Table 1
Ellipse coefficients Q_{ij} from which the a_{ij}'s are obtained.

If r, r' and θ are the object distance, image distance and incidence angle to the normal, respectively, and $a_{20} = (\cos\theta/4)[(1/r) + (1/r')]$, $A = (\sin\theta/2)[(1/r) - (1/r')]$, $C = A^2 + (1/rr')$, then $a_{ij} = a_{20}\,(Q_{ij}/\cos^j\theta)$.

	j						
i	0	1	2	3	4	5	6
0	0	0	1	0	$C/4$	0	$C^2/8$
1	0	0	A	0	$3AC/4$	0	*
2	1	0	$(2A^2 + C)/2$	0	$3C(4A^2 + C)/8$	0	*
3	A	0	$A(2A^2 + 3C)/2$	0	*	0	*
4	$(4A^2 + C)/4$	0	$(8A^4 + 24A^2C + 3C^2)/8$	0	*	0	*
5	$A(4A^2 + 3C)/4$	0	*	0	*	0	*
6	$(8A^4 + 12A^2C + C^2)/8$	0	*	0	*	0	*

most simply by controlling the width, so as to produce a cylinder of arbitrary shape. One can make elliptical cylinders this way as shown by the Berkeley group (Behring *et al.*, 1988; Padmore *et al.*, 1996). Finally one can start with a sphere or cylinder as the initial shape and derive further shapes from that by bending. The most useful of these is a sagittal cylinder bent into a toroid, which, under suitable conditions, can approximate an ellipsoid of revolution. The list of optics that can be made by 100%-contact polishing therefore comprises: flats, spheres, sagittal cylinders and toroids plus tangential cylinders of all shapes including long-radius circular, cubic, elliptical, parabolic *etc.*

Table 2
Toroidal a_{ij}'s (Rah & Howells, 1997).

R and ρ are the major and minor radii of the toroid.

	j						
i	0	1	2	3	4	5	6
0	0	0	$1/(2\rho)$	0	$1/(8R^3)$	0	$1/(16\rho^5)$
1	0	0	0	0	0	0	*
2	$1/(2R)$	0	$1/(4\rho R^2)$	0	$(2\rho + R)/(16\rho^3R^3)$	0	*
3	0	0	0	0	*	0	*
4	$1/(8R^3)$	0	$3/(16\rho R^4)$	0	*	0	*
5	0	0	*	0	*	0	*
6	$1/(16R^5)$	0	*	0	*	0	*

3. Approximating an elliptical cylinder with circular and cubic cylinders

In this section we consider approximation of the elliptical cylinder mirror $X^2/a^2 + Y^2/b^2 = 1$ shown in Fig. 1. The major and minor semiaxes, a and b, and eccentricity e of the ellipse and the coordinates (X_0, Y_0) of the pole of the mirror are related to the optical parameters r, r' and θ by the following equations

$$\begin{aligned} 2a &= r + r' \\ (2ae)^2 &= r^2 + r'^2 - 2rr'\cos 2\theta \\ b^2 &= a^2(1 - e^2) \\ Y_0 &= (rr'\sin 2\theta)/2ae \\ X_0 &= \pm a[1 - (Y_0^2/b^2)]^{1/2}. \end{aligned} \quad (1)$$

In the (x, y) coordinate system, the ellipse can be represented as

$$y = a_2x^2 + a_3x^3 + a_4x^4 + \ldots \quad (2)$$

The a_i's are equivalent to the terms a_{i0} given in Tables 1 and 2 (Rah & Howells, 1997). Each term a_ix^i of the series corresponds to an aberration which will be corrected if the term is faithfully built into the mirror shape. The $i = 2$ term corresponds to defocus, the $i = 3$ one to coma (linear variation of curvature with position), the $i = 4$ one to spherical aberration (quadratic variation of curvature with position) and so on. The quadratic approximation corresponds to building $y = a_2x^2$ and the cubic approximation to building $y = a_2x^2 + a_3x^3$. In some practical situations, such as microprobes and large horizontal collectors, where the mirrors have high demagnification factors, the aberrations can be non-negligible up to very high orders: eight or ten is not unusual. In these cases the quadratic and cubic approximations are expected to be highly inadequate.

In our calculation, the length of the circular or cubic mirror is allowed to extend in each direction until the slope error relative to the ellipse reaches a prescribed value, Δ. The permitted mirror full lengths L_2 and L_3 for the quadratic and cubic approximations, respectively, are thus determined to be

$$\begin{aligned} L_2 &= k_2r'(\Delta/\theta_G)^{1/2} \quad (M \leq 0.5) \\ L_3 &= k_3r'(\Delta/\theta_G)^{1/3} \quad (M \leq 1) \end{aligned} \quad (3)$$

where M is the magnification, θ_G is the grazing angle, and k_2 and k_3 are dimensionless constants with values 3.28 and 2.97, respectively. With an appropriate choice of Δ, the mirrors defined in this way can still achieve any chosen spot size, but their aperture will be limited. When the demagnification factor is high, the aperture restriction will be severe. For condenser mirrors, the requirement is to accept a given beam and deliver a given spot. In this case, equations (3) allow one to decide whether the approximate mirror can meet the requirement or not. For practical synchrotron radiation sources, the answer will often be that it can.

The situation in designing a KB microprobe is quite different. Then we wish to collect as much light as possible and deliver a given spot. To assess how much light-gathering power is lost by approximating the ellipse we have to set an upper limit on the size of the elliptical mirror. This turns out to be (surprisingly) independent of the critical angle and roughly equal to r'. As an example, consider the hard X-ray microdiffraction mirror at the Advanced Light Source, Lawrence Berkeley National Laboratory,

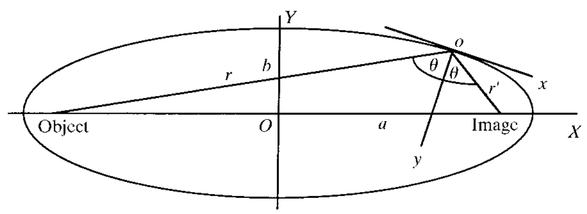

Figure 1
Ellipse layout and notation. The pole of the mirror is at the origin o of the xy coordinate system which is also the point (X_0, Y_0) of the (X, Y) coordinate system.

USA. In the vertical plane, this requires r = 4 m, r' = 0.27 m, θ_G = 3 mrad, Δ = 1.2 µrad (for a desired 0.66 µm spot size). The limits on the lengths of the ellipse, circle and cubic mirrors are thus 270, 18 and 59 mm, respectively. In the horizontal plane, the requirement is r = 4.13 m, r' = 0.13 m, θ_G = 3 mrad, Δ = 2.4 µrad (for the same desired spot size). The limits on the lengths of the ellipse, circle and cubic mirrors in this case are 270, 13 and 37 mm, respectively. Neglecting manufacturing errors, the ratio by which the area of the beam is compressed by the elliptical KB system is 7.2×10^5. The loss of collection area and compression ratio due to using the approximate mirrors is thus a factor 160 for the circle and 16 for the cubic approximation. Even larger losses of X-ray flux are found in corresponding soft X-ray microprobe geometries.

Returning to the question with which we began, it is clear that the choice of building an elliptical cylinder for a condenser mirror or building an approximation depends on the verdict of equations (3). On the other hand, it will always be highly advantageous to build an elliptical cylinder for a microprobe. Moreover, such cylinders can be formed by bending an initially flat mirror that has been polished by a 100%-contact process and does not fall in the class of difficult mirrors (Padmore *et al.*, 1996).

4. Approximating an ellipsoid of revolution with a toroid

The first requirement for a toroid to be an adequate approximation to an ellipsoid of revolution is that the fit should be within tolerance along the tangential center line. This is determined by exactly the same calculation that we completed in the previous section. We now consider the slope errors in the tangential direction (because these always dominate at grazing incidence) at locations close to but not on the tangential center line. To do this we will represent the ellipsoid by a two-dimensional Maclaurin series as follows

$$y = \sum_{i,j} a_{ij} x^i z^j \quad i = 1, 2, 3, \ldots \quad j = 2, 4, \ldots \tag{4}$$

The a_{ij}'s for both an ellipsoid of revolution and toroid are given in Tables 1 and 2. There is a similar expression for a toroid and what we are interested in is the difference between the two. We could represent the difference series by a similar summation over coefficients d_{ij}, say. Now, physically, it is clear that the two surfaces of interest will fit, within a given slope tolerance, over a roughly ellipse-shaped area around the mirror center. We may thus characterize this area by its semiaxes, the major one that we calculate by (3) and the minor one along the z axis that we will calculate now. The tangential slope difference along the z axis is to sixth order

$$(\partial y/\partial x)|_{y=0} = d_{12} z^2 + d_{14} z^4 + \ldots \tag{5}$$

Now the toroid coefficients with indices $(1, j)$ are all zero so we can use the ellipse coefficients instead of the d_{ij}'s and write

$$z_m^2 = [-a_{12} - (a_{12}^2 + 4a_{14}\Delta)^{1/2}]/2a_{14} \tag{6}$$

where z_m is the half width of the area of good fit within tangential slope tolerance Δ.

As an example we could consider an ellipsoidal soft X-ray condenser mirror of 3° grazing angle, distant 16 m from a 100 µm source, imaging at magnification 0.25. This could be approximated by a toroid without significant loss of image quality if the aperture at the mirror was restricted to 9 × 101 mm. At magnification 0.5 the acceptance area would increase about 4.5 times and at magnification unity it would be so large that any conceivable synchrotron radiation beam could be accepted. Toroids also have advantages if a condenser with intentional astigmatism is desired. For example, the radii can be chosen to give a vertical focus at the monochromator entrance slit and a horizontal focus at the exit slit or sample.

5. Discussion and conclusions

We have shown that approximations using easy-to-make mirrors will often make sense for cylinder mirrors designed to image point to point. Equations (3) allows one to determine which cases can benefit from the simplifications involved. Exceptions to this are KB microprobe mirrors where maximal light collection is required. In these cases, it is always advantageous to build elliptical cylinders. Moreover, for both soft and hard X-rays, such elliptical cylinders can generally be built by bending an initially flat mirror. For mirrors where point-to-point imaging by an ellipsoid of revolution appears to be the solution, we have defined the conditions under which a more easily made toroidal surface can provide equal performance. Even when a toroid cannot provide equal performance to a perfectly made ellipsoid, a realistic error budget may still favor it. If the competing ellipsoid is large and specified with a slope tolerance in the arcsecond region, one must talk seriously to potential manufacturers about the challenges and costs of the fabrication that will be required. Sometimes one finds that the choice is between a relaxation of the specification or a best-effort contract. In such cases, the optimum cost–performance trade-off may again shift towards a toroid.

Another approach to replacing a difficult ellipsoid with easier-to-make optics is to use a KB pair. It is true that this involves two mirrors, but the mirrors will be much less expensive and will give the additional flexibility of being able to choose the magnifications separately for the horizontal and vertical directions. This is especially useful for synchrotron radiation sources which normally have different horizontal and vertical sizes.

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