Reciprocal correlation between fringe contrast and amplitude of an anomalous oscillation of X-ray Pendellösung fringes

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As a complement to a previous paper reporting the anomalous fringe oscillation in X-ray Pendellösung interference fringes [Yoshimura (2000), *J. Synchrotron Rad*. **7**, 374–381], this paper presents an amplitude map of the Pendellösung fringe oscillation in order to show that the amplitude distribution has a specific meaningful pattern within the topographic field of Pendellösung fringes. The amplitude distribution is then compared with the distribution of the fringe contrast on a quantitative basis, and it is clarified that the fringe-oscillation amplitude is in good correlation with the reciprocal of fringe contrast. This reciprocal correlation provides evidence that the decrease of fringe contrast from unity is an indirect cause of the anomalous fringe oscillation.

Keywords: X-ray interferometry; Pendellösung fringes; anomalous fringe oscillation; fringe contrast; X-ray topography.

1. Introduction

It has been shown that moiré interference fringes in X-ray topography make a small spatial oscillation along the beam path of diffracted X-rays after emerging from the specimen crystal, when observed by simultaneous imaging onto multi-stacked recording films (Yoshimura, 1987, 1989, 1991, 1996). Such oscillatory non-projective behavior of moiré fringes has not been explained by the current theory of X-ray dynamical diffraction (Kato, 1974; Authier, 1996), and has therefore been thought of as an inexplicable anomaly. Furthermore, it was recently found that a similar anomalous oscillation occurs in equal-thickness Pendellösung interference fringes produced with a simple wedge crystal (Yoshimura, 2000). This discovery added a new view to the prospect of solving the problem, indicating that an anomalous fringe oscillation would be a common basic property of X-ray interference fringes. However, in a previous study of anomalous fringe oscillation an important finding remained unpublished. This is a specificity of the amplitude distribution of the fringe oscillation, leading to a reciprocal correspondence between the oscillation amplitude and the fringe contrast. This finding is expected to be an important clue in the theoretical elucidation of the anomalous fringe oscillation. Here we wish to highlight the experimental observations of the reciprocal correlation. First we describe Pendellösung fringes having a comparatively simple pattern of amplitude and contrast distributions.

2. Anomalous fringe oscillation and its amplitude map

The experimental procedure for observing the anomalous oscillation of Pendellösung fringes has been described in detail in a previous paper (Yoshimura, 2000). Briefly the outline is as follows. The experimental data were obtained as plane-wave (more exactly, quasi-plane-wave) X-ray topographs recorded using synchrotron radiation at KEK-PF. The specimen was a silicon wedge crystal, the X-ray wavelength used was $\lambda_0 = 0.72$ Å, and the 220 reflection was used for the topography. A set of plane-wave diffraction images of the specimen with an equal-thickness Pendellösung fringe pattern was recorded simultaneously onto a set of 10–12 single-coated X-ray films, placed at 52–58 mm from the specimen and being parallel to each other with a mean interval of 0.2–0.6 mm. Topographic images recorded on the multi-stacked films were analyzed using an image analyzer with 8-bit gray levels. The intensity profile of the fringes obtained by the image analysis was studied, and the spatial oscillation of the fringes was derived from the change in shape and position of fringe profiles, seen between member topographs of the multi-film topograph set.

One of 12 Pendellösung topographs of the multi-film topograph set (hereafter T set) treated in this paper is shown in Fig. 1 (member topographs of a T set are numbered in increasing order of specimen-to-film distance as T1, T2 *etc.*). The coordinates (X, Y) are taken on the image plane of the topograph, and the X axis is placed on the black horizontal line in the middle, parallel to the direction of the diffraction vector **g**. The fringe oscillation in the rectangle-enclosed field of size 9.7 mm (X) × 8.0 mm (Y) was analyzed in detail. The fringe spacing in the topograph is almost constant over the whole field, being ~0.34 mm. An example of a topographic observation showing an oscillation in such a fringe pattern among member topographs of the T set was presented in the previous paper (Yoshimura, 2000). An illustration of the fringe profile as the source data for the quantitative analysis of the fringe oscillation, and the resulting fringe-position plot which plainly exhibits the fringe oscillation, was also given in the previous paper. Fringe profiles were measured along the X direction, and the fringe position along the X axis was plotted against the crystal-to-film distance. In the fringe-position plot each fringe oscillates with an amplitude and a frequency which appear to be characteristic of the relevant fringe, although on the other hand the amplitude and the frequency are not very well fixed even within the oscillation of one fringe. For such a fringe oscillation, the oscil-

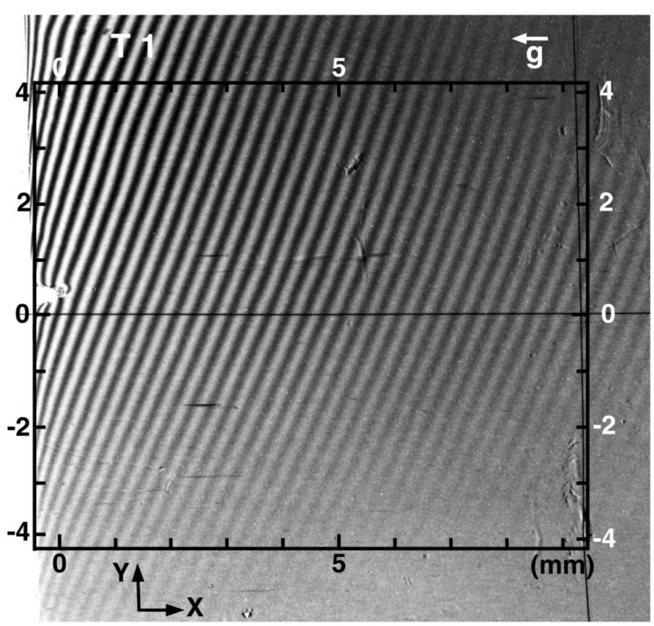

Figure 1
One of the multi-film Pendellösung topographs studied in this paper. T set No. 4, T1 topograph, *O*-wave image. **g** is the diffraction vector. White contrast indicates higher intensity. The scales along the X and Y axes are graduated for the real dimension on the exit surface of the specimen. The black horizontal line in the middle and the crossing vertical line near the right-hand edge are shadows of thin Pt lines placed between the specimen and the recording films, used as a reference of position and orientation on the topographs.

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lation amplitude, a_P, was practically evaluated by the formula $a_P = 2^{1/2}\sigma$, where σ is the standard deviation of the respective fringe-position plots. The amplitude a_P thus determined is considered to be a good measure for representing the magnitude of oscillation.

Observations of fringe profiles and fringe-position plots as above at various Y positions suggest that the amplitude of fringe oscillation follows some simple-patterned distribution rule. To clarify this, extensive image analysis was performed on all the member topographs over the entire topographic field. Fig. 2 presents the resulting amplitude map. In this diagram, short horizontal bars along the X axis indicate the oscillation range $\pm a_P$ centered on the mean fringe position. As expected, the amplitude map shows a simple location dependence of the present anomalous fringe oscillation. The amplitude a_P is at a negligible level (0.3–0.7 pixels) in the upper left-hand quarter of the field, rises slowly from this error level in the area $X \geq 5.0$ and/or $Y \leq 0.0$, and reaches a maximum of 6–7 pixels, 33–38% of the fringe spacing, in the lower right-hand corner. This maximum value is roughly the same as that of the previous moiré fringes (Yoshimura, 1996). The fringe pattern ceases to show contrast around the lower right-hand corner, though the present blank in the map is caused by a disturbance to the normal fringe image by a defect image originating from the monochromators upstream.

3. Comparison between oscillation amplitude and fringe contrast

3.1. Overall comparison

As may be clearly seen, an overall comparison of Figs. 1 and 2 suggests that there is some correspondence between the Pendellösung topograph and the amplitude map. Let us examine in more detail the suggested correspondence with respect to two factors in the topograph, *i.e.* the image intensity and the fringe contrast. The peak or the maximum intensity of fringes decreases monotonically towards the $+X$ direction, showing an apparent reciprocal correspondence with the amplitude map, but keeps an almost constant level or slightly increases downward in the Y direction. Such an intensity distribution does not match the amplitude map directly or reciprocally. The bottom or the minimum intensity of fringes, on the other hand, slightly increases or decreases towards the $+X$ direction, and increases more rapidly towards the $-Y$ direction, with no consistent correspondence with the amplitude map. In contrast, the fringe contrast decreases monotonically from the upper left-hand to the lower right-hand corner on the whole, showing a consistent reciprocal correspondence with the amplitude map in either of the X and Y directions.

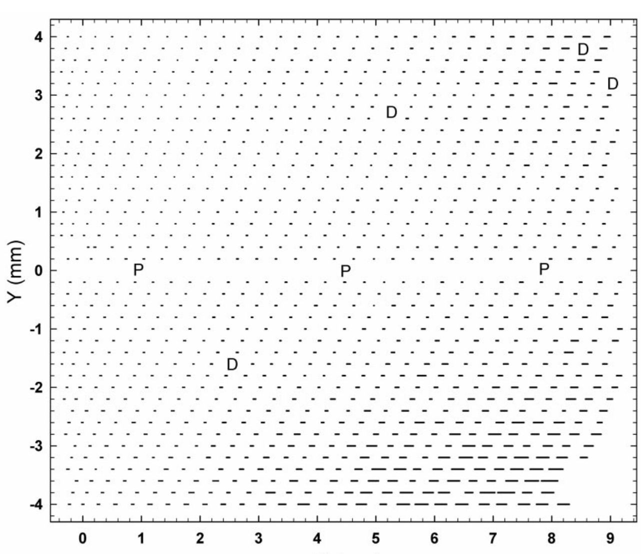

Figure 2
Amplitude map of the anomalous oscillation of Pendellösung fringes over the whole field of interest. T set No. 4, *O*-wave image. *P* and *D* indicate sites at which the fringe position was not determined correctly because of disturbances in the fringe profile, originating from the Pt-line shadows and point-like defect images, respectively.

3.2. Reciprocal correlation between oscillation amplitude and fringe contrast

In order to confirm the suggested reciprocal correlation between fringe contrast and oscillation amplitude, we performed a microdensitometric measurement of the data topographs in order to evaluate the fringe contrast exactly. New measurements were made using a microdensitometer (Konica PDM-5 type B), aside from the previous densitometric measurements (Yoshimura, 2000). The photometric slit size was 20 μm (width) × 100 μm (height) on the sample film. The densitometric scan was made along the X direction on the topographs, similar to the image-analyzer intensity scan. The measured photographic density, D, was 0.30–1.19 including a fog density of ~0.15 over the entire topographic field. The fringe contrast, V, was evaluated from the maximum and the minimum D values of fringes, according to the usual formula of fringe contrast (Yoshimura, 2000). Although the D value can be transformed into the image intensity, I, of topographs using an empirical equation (Yoshimura, 1993), in order to evaluate fringe contrast as exactly as possible, the re-evaluation resulted in no appreciable difference. In the comparison with the oscillation amplitude below, the fringe contrast value measured in the T1 topograph was used as the representative.

Fig. 3 shows a correlation diagram with respect to the oscillation amplitude and the fringe contrast. Oscillation amplitude, a_P/L, relative to the fringe spacing, L, is plotted against fringe contrast, V, for 265 measured points in total on the horizontal lines at every 1 mm from Y = 4.0 to Y = −4.0 in the amplitude map in Fig. 2. Here the fringe spacing L is averaged over all the 12 member topographs of the T set, for the respective measured points in the amplitude map. For

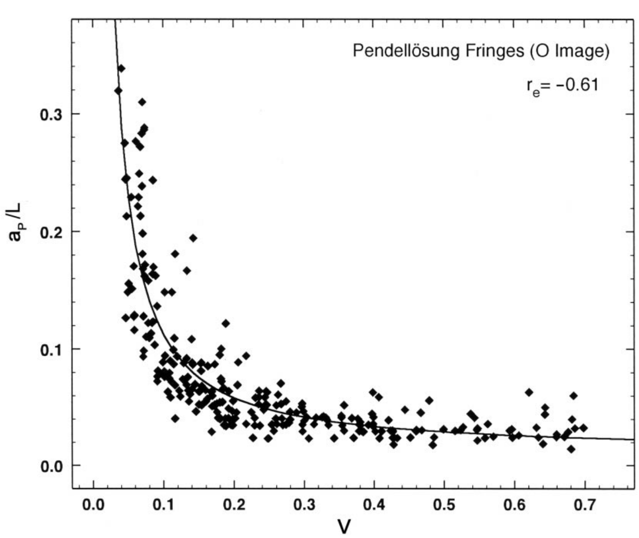

Figure 3
Correlation diagram with respect to fringe-oscillation amplitude a_P/L and fringe contrast V of the Pendellösung fringes. The number in the upper right-hand corner gives the correlation coefficient.

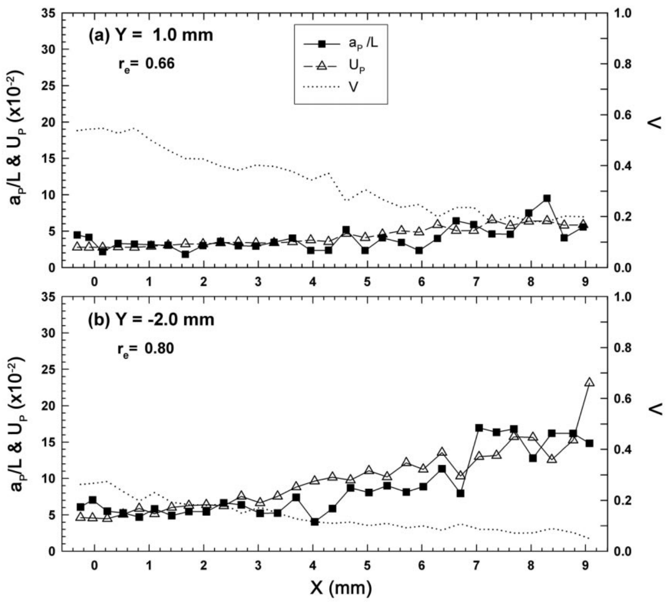

Figure 4
Fringe-by-fringe plots of oscillation amplitude a_P/L and modified reciprocal contrast U_P along the X axis on the lines at (a) Y = 1.0 and (b) $Y = -2.0$.

the oscillation amplitude, a_P/L is used here rather than a_P, though either can be used with no important difference in the present Pendellösung fringes, having almost constant fringe spacing. The presented correlation diagram clearly shows that the amplitude a_P/L and the fringe contrast V are correlated, forming a hyperbolic curve. The computed fitted curve was given by

$$a_P/L = 0.0075V^{-1.1} + 0.013. \quad (1)$$

We assumed a simple-formed equation with no preconception for this curve fitting.

Fig. 4 shows a fringe-by-fringe comparison between the oscillation amplitude and the fringe contrast for fringes at different Y positions. For the fringe contrast against the relative amplitude a_P/L, the quantity U_P given by the right-hand side of (1) (*i.e.* $U_P \equiv 0.0075V^{-1.1}$ + 0.013) is plotted together with the unmodified contrast V for reference. The correlation coefficient, r_e, of the correlation within the respective graph is given in the upper left-hand corner in each graph. Although already evident in the correlation diagram in Fig. 3, Figs. 4(a) and 4(b) again show that the amplitude a_P/L and the modified reciprocal contrast U_P are in good correlation with each other over the whole topographic field. It should be noted, however, that, in spite of the good correlation on the whole, the correlation does not mean a tight-binding correspondence held in every fringe with the same closeness. In addition to the oscillation of the fringes, the oscillation amplitude and reciprocal contrast oscillate locally in their distributions. The local oscillations of the two quantities match in some fringes but fail to match in other fringes. This relaxed correspondence is responsible for the moderated values of the correlation coefficient, and shows characteristics of the present correspondence.

The correlation coefficient for all the mentioned 265 measured points between a_P/L and U_P was r_e = 0.86. According to the analysis of testing statistical hypotheses and statistical estimation, a population correlation exists between the two quantities with a probability better than 99.9%, and the 99.9% confidence interval of the population correlation coefficient, ρ_e, is $0.79 < \rho_e < 0.91$.

4. Conclusions

The extensive and detailed analysis of experimental data described has clearly shown that the amplitude of anomalous fringe oscillation is in good reciprocal correlation with the distribution of fringe contrast. The relatively relaxed correspondence between oscillation amplitude and fringe contrast in spite of the good reciprocal correlation on the whole may be interpreted as showing that the fringe contrast does not directly control the oscillation behavior of fringes, but works behind the oscillation to regulate the environmental condition for the oscillation to occur. In this sense the decrease of fringe contrast from unity may be considered to be an indirect cause of the amplification, and of the occurrence itself, of the anomalous fringe oscillation.

The observation in Fig. 4 confirms that U_P is practically a good expression of modified reciprocal contrast. In addition, curve-fitting trials in Fig. 3 show that an expression of the form $a(V^{-1} - 1)^m + b$ can also be a good expression of modified reciprocal contrast. To approach the true form of modified reciprocal contrast, oscillation amplitude and fringe contrast need to be measured over a wider range with better accuracy.

It should be mentioned that the fringe-oscillation amplitude correlates not with the fringe contrast produced when an ideal plane wave is incident on the crystal but with the practically observed fringe contrast approximating to the calculated contrast for the incident wave with a angular width of 0.34 arcsec [see Fig. 4 in Yoshimura (2000)]. As can easily be shown by calculation, fringe contrast greatly differs between the two cases where the ideal plane wave is incident and where the slightly divergent wave is incident.

The experiment for the present Pendellösung-fringe study was conducted under the approval of the Photon Factory Program Advisory Committee (Proposal No. 90-113).

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