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Synchrotron radiation loss of a laser accelerator based on an inverse electron cyclotron resonance maser

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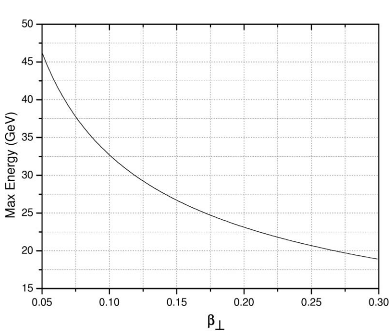
A laser accelerator based on an electron cyclotron resonance maser is described. Important losses that give rise to saturation length and saturation power are included in the analysis. The results are compared with results from an inverse free-electron laser accelerator.



1. Introduction

The field strength in a typical RF accelerating cavity is considerably smaller than that which can be achieved in modern advanced lasers. This leads to growing interest in laser accelerators that produce charged particle beams at highly relativistic energies using intense coherent electromagnetic waves. In the past, various mechanisms and accelerator schemes have been proposed and adopted for laser accelerators. Successful experiments on free-electron lasers have seen the development of inverse free-electron laser (IFEL) accelerators (Palmer, 1972; Courant *et al.*, 1985; Musumeci *et al.*, 2005; Duris *et al.*, 2012, 2014; Khullar *et al.*, 2015), achieving energy transfer from laser beams to electrons. In the IFEL scheme, the relativistic electrons move through an undulator magnet collinearly with an electromagnetic wave. The undulator magnet produces a small transverse velocity in the direction parallel to the electric-field vector of the collinear propagating electromagnetic wave, so that energy transfer between the electron and the electromagnetic wave is possible. The undulator was tapered using an IFEL scheme in order to accelerate electrons to energies of the order of GeV (Sprangle & Tang, 1981; Pellegrini, 1982). An alternative scheme has been developed that involves the laser acceleration of electrons in a uniform magnetic field based on the electron cyclotron resonance maser principle (Colson & Ride, 1979; McDermott *et al.*, 1985; Shpitalnik *et al.*, 1991, 1992; Chen, 1991, 1992; Sprangle *et al.*, 1983; Lie, 2002; Mirzanejhad *et al.*, 2006). In this scheme, an electron with an initial perpendicular velocity component is guided by the uniform axial field. Initially, the electrons undergo cyclotron motion and they are uniformly arranged in a rotational phase on the circle. The Larmar radius is defined as $r_{\rm L} = v_{\perp}/\omega_{\rm c}$, where $v_{\perp}$ is the initial perpendicular velocity of the electron and $\Omega_{\rm c} = eB/m\gamma$ is the relativistic cyclotron frequency. After interaction with the electromagnetic fields, some electrons that are decelerated in the electric field rotate faster to accumulate phase lead. Some electrons that are accelerated rotate slowly to accumulate phase lag. This phase bunching leads to a coherent radiation mechanism of the electron cyclotron resonance maser. The energy transfer is proportional to that produced, $v_\perp E_\perp$, where $v_\perp$ is the electron transverse velocity component and $E_\perp$ is the transverse electric field.

In this paper, we consider laser acceleration of electrons based on the inverse electron cyclotron resonance maser principle. We consider cyclotron radiation loss as an important factor for the net acceleration of the laser accelerator and we include this in our analysis. We solve relativistic Lorentz force equations in a uniform axial magnetic field, assuming circularly polarized electromagnetic waves of constant amplitude. An analytical solution has been obtained for the maximum energy attained in the laser accelerator. The solutions yield a dependence of the electron perpendicular velocity component on the maximum energy attainable, the saturation length and the saturation energy of the laser accelerator. It has been shown that in a magnetic field of 16 T, power of 400 GW, a 10.6 µm wavelength laser with an optimum electron beam with a Larmar radius in the range 10–30 µm can accelerate electrons to ∼20 GeV in less than 5 m.

2. Accelerator equations

The particle motion and energy change of a single charged particle in the presence of a uniform axial magnetic field and a laser are governed by relativistic Lorentz force equations. We consider a uniform magnetic field produced by a solenoid and given by

$$(0, 0, \mathbf{B}_0).$$

An electromagnetic wave of a circularly polarized laser is described by

$$\begin{aligned} E_\mathrm{r}(z,t) &= E_0\big[\cos(k_\mathrm{r}z-\omega_\mathrm{r}t),\, \sin(k_\mathrm{r}z+\omega_\mathrm{r}t),\, 0\big],\\ B_\mathrm{r}(z,t) &= E_0\big[\sin(k_\mathrm{r}z+\omega_\mathrm{r}t),\, \cos(k_\mathrm{r}z+\omega_\mathrm{r}t),\, 0\big]. \end{aligned} \tag{1}$$

Here, k_r and ω_r denote the wavenumber and the frequency of the electromagnetic wave, respectively. The equation describing the motion of the electrons in the IFEL can be derived from the Lorentz equation of motion,

$$\frac{\mathrm{d}(\boldsymbol{\beta})}{\mathrm{d}t} = \frac{e}{m_\mathrm{e}c\gamma}\big[(\mathbf{E}_\mathrm{r}) + \boldsymbol{\beta}(\mathbf{B}_\mathrm{r}+\mathbf{B})\big]. \tag{2}$$

In component form, equation (2) reads

$$\begin{aligned} \frac{\mathrm{d}\boldsymbol{\beta}_x}{\mathrm{d}t} &= \Omega_\mathrm{L}(1-\beta_z)\cos\xi + \Omega_\mathrm{c}\beta_y,\\ \frac{\mathrm{d}\boldsymbol{\beta}_y}{\mathrm{d}t} &= -\Omega_\mathrm{L}(1-\beta_z)\sin\xi - \Omega_\mathrm{c}\beta_x, \end{aligned} \tag{3}$$

where $\Omega_\mathrm{L} = eE_0/mc\gamma, \Omega_\mathrm{c} = eB_0/mc\gamma$ and $\xi = k_\mathrm{r}z - \omega_\mathrm{r}t$. Equation (3) for the electron velocity is read as

$$\begin{aligned} \frac{\mathrm{d}^2\beta_x}{\mathrm{d}t^2} + \Omega_\mathrm{c}^2\beta_x &= \Omega_\mathrm{L}(1-\beta_z)[\omega_\mathrm{r}(1-\beta_z)-\Omega_\mathrm{c}]\sin\xi,\\ \frac{\mathrm{d}^2\beta_y}{\mathrm{d}t^2} + \Omega_\mathrm{c}^2\beta_y &= \Omega_\mathrm{L}(1-\beta_z)[\omega_\mathrm{r}(1-\beta_z)-\Omega_\mathrm{c}]\cos\xi, \end{aligned} \tag{4}$$

and the solutions are

$$\begin{aligned} \beta_x &= \beta_\perp\sin\Omega_\mathrm{c}t - \frac{\Omega_\mathrm{L}(1-\beta_z)}{\omega_\mathrm{r}(1-\beta_z)+\Omega_\mathrm{c}}\sin\xi,\\ \beta_y &= -\beta_\perp\cos\Omega_\mathrm{c}t - \frac{\Omega_\mathrm{L}(1-\beta_z)}{\omega_\mathrm{r}(1-\beta_z)+\Omega_\mathrm{c}}\cos\xi. \end{aligned} \tag{5}$$

We write equation (5) as

$$\beta_x = \beta_\perp\sin\Omega_\mathrm{c}t + \Omega_\mathrm{L}'\sin\xi, \tag{5a}$$

$$\beta_y = -\beta_\perp\cos\Omega_\mathrm{c}t - \Omega_\mathrm{L}'\cos\xi, \tag{5b}$$

where

$$\Omega_\mathrm{L}' = \frac{\Omega_\mathrm{L}(1-\beta_z)}{\omega_\mathrm{r}(1-\beta_z)+\Omega_\mathrm{c}}.$$

Substituting β_x and β_y from equations (5*a*) and (5*b*) into the relation $\beta^2 = 1 - 1/\gamma^2$, we obtain the longitudinal velocity,

$$\beta_z = 1 - \frac{1}{2\gamma^2}\big[1 + \gamma^2\beta_\perp^2 + \gamma^2\Omega_\mathrm{L}'^2 + 2\gamma\beta_\perp\gamma\Omega_\mathrm{L}'\cos\psi\big], \tag{6}$$

where $\psi = k_\mathrm{r}z - \omega_\mathrm{r}t + \Omega_\mathrm{c}t$. The longitudinal coordinate can be evaluated by integrating the above expression to obtain

$$z = \left[1 - \frac{1}{2\gamma^2}\left(1 + \gamma^2\beta_\perp^2 + \gamma^2{\Omega_\mathrm{L}'}^2\right)\right]ct - \frac{\beta_\perp\Omega_\mathrm{L}'}{\omega_\mathrm{r}(\beta_z - 1)+\Omega_\mathrm{c}}\sin\psi. \tag{7}$$

The change in electron energy is given by

$$\frac{\mathrm{d}\gamma}{\mathrm{d}t} = \frac{e}{m_\mathrm{e}c}\mathbf{E}\boldsymbol{\beta}. \tag{8}$$

Using equation (5), the change in electron energy is given by

$$\frac{\mathrm{d}\gamma}{\mathrm{d}t} = Ac\beta_\perp\sin\psi, \tag{9}$$

where $A = eE_0/m_ec^2$ and the phase $\psi = k_\mathrm{r}z - \omega_\mathrm{r}t + \Omega_\mathrm{c}t$. The resonant condition for $\mathrm{d}\psi/\mathrm{d}z = 0$ is

$$\Omega_\mathrm{c} = \frac{\omega_\mathrm{r}}{2\gamma^2}\big[1 + \gamma^2\beta_\perp^2\big]. \tag{10}$$

In an accelerator, the effects of synchrotron-radiation loss are important and give the saturation length and saturation power of the device,

$$\frac{\mathrm{d}\gamma}{\mathrm{d}t} = \frac{e}{m_\mathrm{e}c}\mathbf{E}\boldsymbol{\beta} - \frac{1}{m_\mathrm{e}c^2}\frac{\mathrm{d}P}{\mathrm{d}t}, \tag{11}$$

where

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{2}{3}\frac{e^2}{c}\gamma^6\left[\dot{\beta}^2 - (\beta\times\dot{\beta})^2\right]. \tag{12}$$

and $\dot{\beta} = \mathrm{d}\beta/\mathrm{d}t$. Using equation (5), the synchrotron-radiation loss of the electron is given by

$$\frac{1}{m_\mathrm{e}c^2}\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{2}{3}r_\mathrm{e}\frac{\gamma^6}{c}\beta_\perp^2\Omega_\mathrm{c}^2, \tag{13}$$

where $r_\mathrm{e} = e^2/m_\mathrm{e}c^2$ is the classical electron radius. We can write the accelerator equation, including the synchrotron loss term, as

$$\frac{\mathrm{d}\gamma}{\mathrm{d}t} = Ac\beta_\perp\sin\psi - \frac{2}{3}r_\mathrm{e}\frac{\gamma^6}{c}\beta_\perp^2\Omega_\mathrm{c}^2. \tag{14}$$

Using the resonant condition in equation (14), $\Omega_c \simeq \omega_r/2\gamma^2$. The accelerator equation becomes

$$\frac{\mathrm{d}\gamma}{\mathrm{d}t} = Ac\beta_\perp \sin\psi - \frac{1}{6} r_e \gamma^2 c \beta_\perp^2 k_r^2. \tag{15}$$

Using equation (15) for the undulator length, we obtain

$$\frac{\mathrm{d}\gamma}{\mathrm{d}z} = A\beta_\perp \sin\psi - \frac{1}{6} r_e \gamma^2 \beta_\perp^2 k_r^2. \tag{16}$$

The maximum energy attainable from the device ($\gamma \to \gamma_\infty$) occurs when we set $\mathrm{d}\gamma/\mathrm{d}z = 0$. This gives

$$\gamma_\infty = \left[\frac{3\tilde{A}\lambda_r^2}{2\pi^2 r_e \beta_\perp}\right]^{1/2}. \tag{17}$$

where $\tilde{A} = A\sin\psi$. Equation (16), in terms of the maximum attainable energy, can be written as

$$\frac{\mathrm{d}\gamma}{\mathrm{d}z} = A\beta_\perp \sin\psi \left[1 - \frac{\gamma^2}{\gamma_\infty^2}\right]. \tag{18}$$

Using the standard integral results,

$$\int \frac{\mathrm{d}x}{1 - bx^2} = \frac{1}{2b} \ln\left|\frac{1+bx}{1-bx}\right|,$$

the solution for equation (18) is written as

$$\gamma(z) = \gamma_\infty - (\gamma_\infty - \gamma_0)\exp\left[-\frac{2A\beta_\perp \sin\psi}{\gamma_\infty}(z - z_0)\right], \tag{19}$$

where γ_0 is the initial energy of the electron.

3. Results and discussion

A laser accelerator based on inverse electron cyclotron resonance maser is described. The effects of synchrotron radiation losses are included in the analysis. The relativistic Lorentz force equation is solved analytically for an electron beam having an initial finite perpendicular velocity. The electron beam travels in a helical path in the axial magnetic field and exchanges energy with a copropagating circularly polarized laser field. The electric field of the laser can be calculated using

$$E\,[\mathrm{V\,m^{-1}}] = \frac{27.5\,W^{1/2}}{\pi^{1/2}\,w_0},$$

where W is the laser power and w_0 is the laser spot size. A 400 GW, 10.6 µm-wavelength laser focused to a 240 µm spot size gives an electric field strength of E = 1.33×10^6 statV cm^{-1} [1 V = (1/3) × 10^{-2} statV]. The maximum energy attainable from the laser accelerator can be estimated from equation (17). Considering r_e = 2.8179 × 10^{-13} cm, A = 771 cm^{-1} and $\sin\psi$ = 0.866, we rewrite equation (17) as

$$\gamma_\infty = 200.5 \times 10^2 (1/\beta_\perp)^{1/2}.$$

For $\beta_\perp = 0.2$, $\gamma_\infty = 44833$ and the maximum attainable energy from the accelerator is $E_\infty(\gamma_\infty mc^2)$ = 23 GeV (Fig. 1).

The maximum attainable energy in the laser accelerator is inversely proportional to the square root of the perpendicular velocity, *i.e.* $\gamma_\infty \propto (1/\beta_\perp)^{1/2}$. An electron beam with a lower $\beta_\perp$ can provide high energy but with a lower gradient [equation (17)]. This implies that a lower value of $\beta_\perp$ would require a large saturation length and is therefore undesirable. A higher value of $\beta_\perp$ implies efficient interaction with the laser and the saturation length is smaller and can result in a compact laser accelerator device. The energy gain curves are drawn in Fig. 2 for γ_0 = 30 and for a $\beta_\perp$ value in the range $\beta_\perp$ = 0.1–0.3. Higher $\beta_\perp$ values reach saturation quickly but the saturation energy is substantially lower. The saturation length and saturation energy are plotted in Figs. 3 and 4, respectively; for $\beta_\perp$, the values are 5 m and ~21 GeV. To estimate the analytical expression for the acceleration gradient, we consider equation (16) without synchrotron-radiation losses, *i.e.*

$$\frac{\mathrm{d}E}{\mathrm{d}z} = mc^2 A\beta_\perp \sin\psi.$$

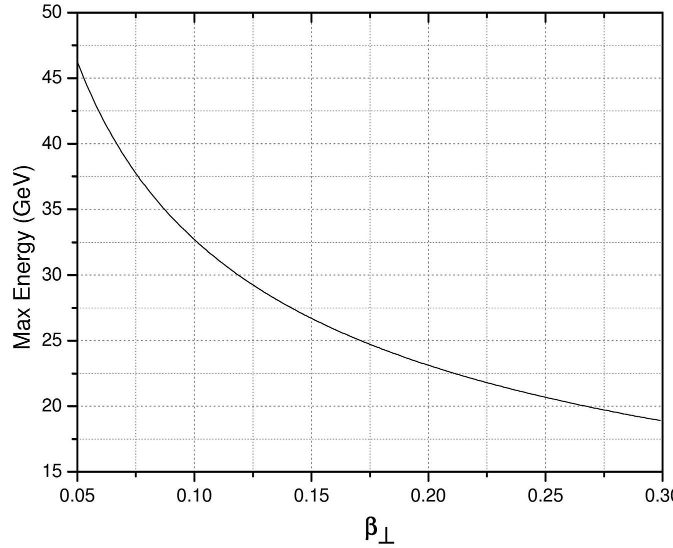


Figure 1
Maximum energy *versus* $\beta_\perp$.

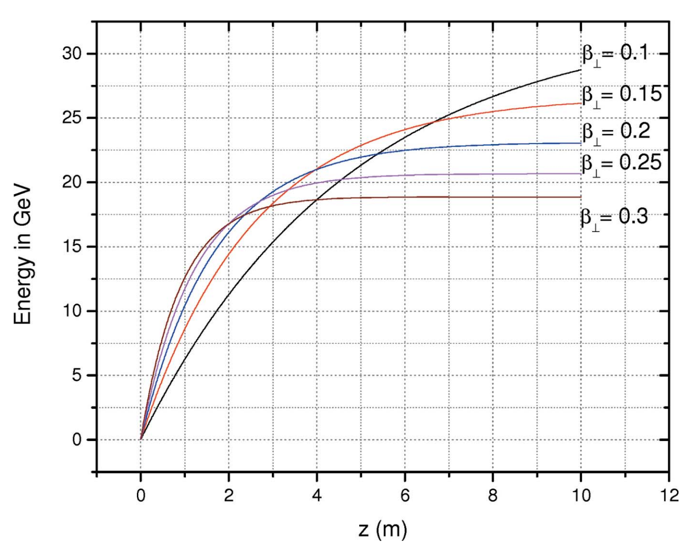


Figure 2
Energy of electrons *versus* distance for ψ = 120°.

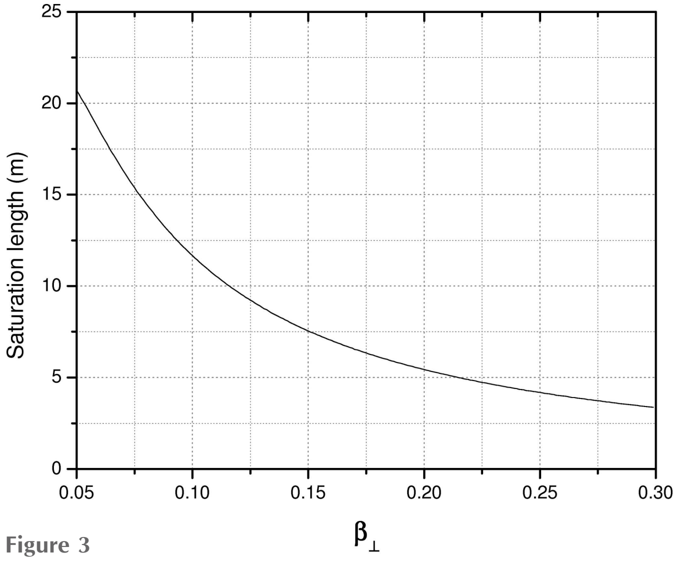


Figure 3
Saturation length *versus* $\beta_\perp$.

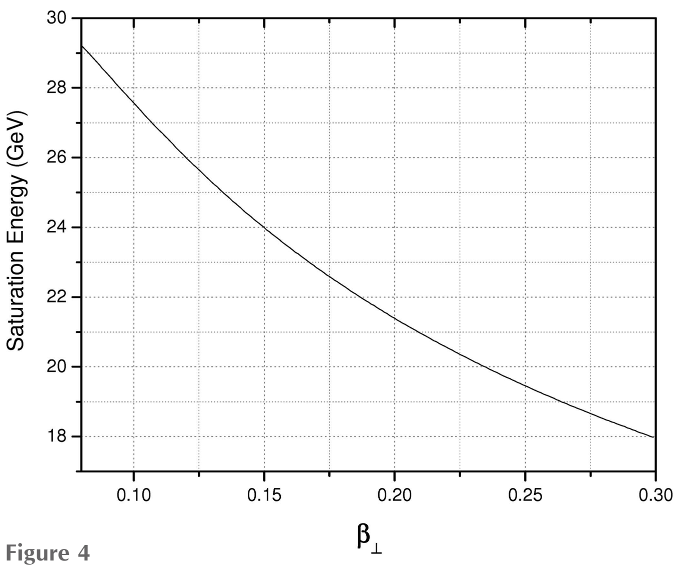


Figure 4
Saturation energy *versus* $\beta_\perp$.

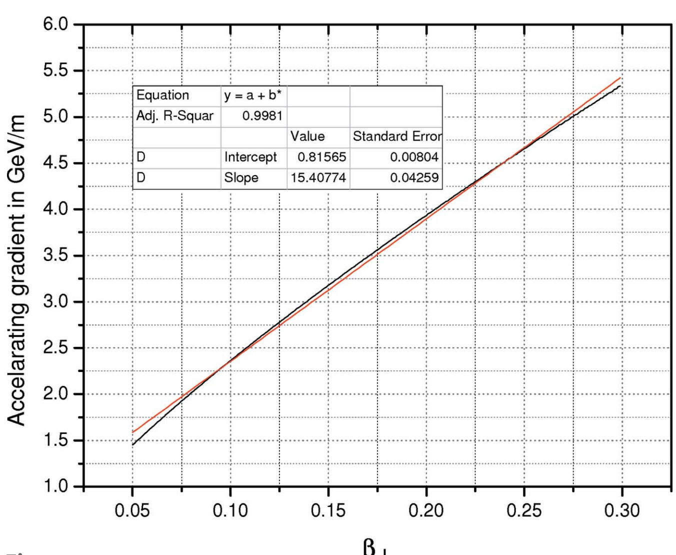


Figure 5
Accelerating gradient *versus* $\beta_\perp$.

For mc^2 = 0.511 MeV, A = 771 cm^{-1} and $\sin\psi$ = 0.866, we rewrite the equation in practical units as

$$\frac{\mathrm{d}E}{\mathrm{d}z} = 34\,\beta_\perp \quad [\mathrm{GeV\,m^{-1}}].$$

For $\beta_\perp = 0.2$, the accelerating gradient is 6.8 GeV m^{-1}. Using the resonant condition from equation (10) (*i.e.* $\Omega_c = \omega_r/2\gamma^2$), the magnetic field in our calculation is 16.8 T for γ_0 = 30. In Fig. 5 we calculated the accelerating gradient. A linear fit for the calculation yields

$$\text{gradient}\ [\mathrm{GeV\,m^{-1}}] = 0.8156 + 15.4077\beta_\perp.$$

For $\beta_\perp$ = 0.2, the calculation gives 3.93 GeV m^{-1}, whereas the fit predicts a value of 3.91 GeV m^{-1}. The discrepancy between the calculation and the fit formula is almost 2%. The Larmar radius of the electron at this value can be calculated from

$$r_L = c\beta_\perp/\omega_c.$$

For $\beta_\perp$ in the range 0.1–0.3, the Larmar radius will be in the range 10–30 µm.

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