JOURNAL OF SYNCHROTRON RADIATION

ISSN 1600-5775

Received 29 September 2014
Accepted 22 January 2015

Edited by D. A. Reis, SLAC National Accelerator Laboratory, USA

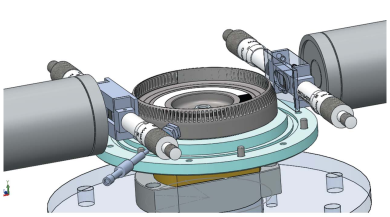
Keywords: Mössbauer effect; time dilation.

Synchrotron radiation Mössbauer spectra of a rotating absorber with implications for testing velocity and acceleration time dilation

Y. Friedman,[a]* E. Yudkin,[a] I. Nowik,[b] I. Felner,[b] H.-C. Wille,[c] R. Röhlsberger,[c] J. Haber,[c] G. Wortmann,[d] S. Arogeti,[e] M. Friedman,[e] Z. Brand,[f] N. Levi,[f] I. Shafir,[f] O. Efrati,[g] T. Frumson,[g] A. Finkelstein,[g] A. I. Chumakov,[h] I. Kantor[h] and R. Rüffer[h]

[a]Jerusalem College of Technology, POB 16031, Jerusalem 91160, Israel, [b]Racah Institute of Physics, Hebrew University, Jerusalem 91904, Israel, [c]Deutsches Elektronen-Synchrotron, Notkestrasse 85, D-22607 Hamburg, Germany, [d]Department Physik, Universität Paderborn, Warburger Strasse 100, D-33098 Paderborn, Germany, [e]Ben-Gurion University of the Negev, Ber Sheva, Israel, [f]Nuclear Research Center Negev, Ber Sheva, Israel, [g]Colibri Spindles Ltd, Industrial Park Lavon, Bdg 1, MP Bikat Bet Hakerem 2011800, Israel, and [h]European Synchrotron Radiation Facility, BP 220, F-38043 Grenoble, France. *Correspondence e-mail: friedman@jct.ac.il

Many Mössbauer spectroscopy (MS) experiments have used a rotating absorber in order to measure the second-order transverse Doppler (TD) shift, and to test the validity of the Einstein time dilation theory. From these experiments, one may also test the clock hypothesis (CH) and the time dilation caused by acceleration. In such experiments the absorption curves must be obtained, since it cannot be assumed that there is no broadening of the curve during the rotation. For technical reasons, it is very complicated to keep the balance of a fast rotating disk if there are moving parts on it. Thus, the Mössbauer source on a transducer should be outside the disk. Friedman and Nowik have already predicted that the X-ray beam finite size dramatically affects the MS absorption line and causes its broadening. We provide here explicit formulas to evaluate this broadening for a synchrotron Mössbauer source (SMS) beam. The broadening is linearly proportional to the rotation frequency and to the SMS beam width at the rotation axis. In addition, it is shown that the TD shift and the MS line broadening are affected by an additional factor assigned as the *alignment shift* which is proportional to the frequency of rotation and to the distance between the X-ray beam center and the rotation axis. This new shift helps to align the disk's axis of rotation to the X-ray beam's center. To minimize the broadening, one must focus the X-ray on the axis of the rotating disk and/or to add a slit positioned at the center, to block the rays distant from the rotation axis of the disk. Our experiment, using the ^{57}Fe SMS, currently available at the Nuclear Resonance beamline (ID18) at the ESRF, with a rotating stainless steel foil, confirmed our predictions. With a slit installed at the rotation axis (reducing the effective beam width from 15.6 μm to 5.4 μm), one can measure a statistically meaningful absorption spectrum up to 300 Hz, while, without a slit, such spectra could be obtained up to 100 Hz only. Thus, both the broadening and the alignment shift are very significant and must be taken into consideration in any rotating absorber experiment. Here a method is offered to measure accurately the TD shift and to test the CH.



1. Introduction

After the discovery of the Mössbauer effect in 1958, quantitative measurements of relativistic time dilation, expressed by the transverse Doppler (TD) shift, were carried out in the 1960s (Hay *et al.*, 1960; Hay, 1962; Cranshaw *et al.*, 1960; Cranshaw & Hay, 1963; Champeney & Moon, 1961; Champeney *et al.*, 1965; Kündig, 1963). All of these experiments reported full agreement with the time dilation predicted by Einstein's theory of relativity. In the experiments, except that of Kündig (1963), the Mössbauer source was placed at the center of a fast rotating disk and an absorber at the rim of the disk. In the analysis of these experiments, it was assumed that the absorption line of the rotating absorber has a Lorentzian shape with the same width as at rest and its shift is caused by the time dilation factor only.

As it was shown in Kündig's experiment, there is a broadening of the absorption curves during the rotation, and the shift of the curve cannot be derived by measuring one point of the spectrum. Kholmetskii *et al.* (2009) tested time dilation, measured by the transverse Doppler shift, with a Mössbauer source, placed at the center of a fast rotating disk, and the use of two Mössbauer absorbers. They used one absorber to estimate the broadening and to correct the data from the second absorber. They observed a deviation from relativity predictions. Note that since a rotating absorber is significantly accelerated, these experiments could be used to test the clock hypothesis of Einstein by measuring the shift between the absorption spectra of two sides of a rotating Mössbauer absorber.

Usually, absorption spectral lines are obtained by putting a Mössbauer source on a transducer and measuring the transmission at different velocities, which by first Doppler shift is translated to transmission at different frequencies near the resonant one. For technical reasons, it is very complicated to keep the balance of a fast rotating disk, if there are moving parts on it. Thus, we are forced to have the transducer outside the rotating disk and to ensure that the rotations are vibrationless. Friedman & Nowik (2012) claimed that the absorption line of a rotating Mössbauer absorber becomes broader during the rotation for a source outside the rotating disk. This is due to the fact that the beam must have a finite width. Therefore, the velocity of the absorber is not always perpendicular to all individual rays. Hence, these rays undergo a longitudinal Doppler shift in addition to the expected transversal one. This shift is very significant even for very narrow beams.

Based on the generalized principle of relativity and the ensuing symmetry, it was shown by Friedman & Gofman (2010) that there are only two possible types of transformations between uniformly accelerated systems. The validity of the clock hypothesis is crucial for determining which one of the two types of transformations is valid. This hypothesis, as stated by Einstein (1911), claims that the rate of an accelerated clock is equal to that of a co-moving unaccelerated clock. If the clock hypothesis is not true, then a universal maximal acceleration a_m exists and, as predicted (Friedman, 2011), a Doppler type shift due to acceleration will be observed. This Doppler type shift is similar to the Doppler shift due to the velocity of the source, and has the same formula, but v/c is replaced by a/a_m.

We show here that the absorption line broadening depends on the width of the X-ray beam at the center of the disk. Therefore, in order to minimize the broadening, one must focus the X-ray on the axis of the rotating disk and/or add a slit positioned at the center, to block the rays distant from the rotation axis of the disk.

We obtained explicit formulas for this broadening by using the following assumptions. (i) We assume a converging beam with a focal spot localized approximately at the rotation axis of the disk; the beam front has a normalized Lorentzian distribution around position b_0 (distance from the beam to the rotation axis) and half width d. (ii) The rotating absorber has a single narrow line spectrum with half width γ_0 and relative absorption amplitude A_0. We show that under these assumptions the absorption spectra at rotation angular velocity ω_r will be approximately Lorentzian with a new shift, the alignment shift $\omega_r b_0$, in addition to the known TD shift, with half width $\gamma_r = \gamma_v + \omega_r d$ and relative absorption amplitude $A_r = A_0\gamma_v/\gamma_r$, where $\gamma_v = c\gamma_0/\omega_0$ and ω_0 is the resonant frequency of the Mössbauer absorber. We can use the alignment shift for alignment of the rotating system, namely to move the rotating system to the position $b_0 = 0$.

These predictions were tested recently at beamline ID18 of the European Synchrotron Radiation Facility (ESRF) by using a specially designed rotor system and the recently invented synchrotron Mössbauer source (SMS) (Potapkin *et al.*, 2012). KB optics (Kirkpatrick & Baez, 1948) were used to focus the X-rays to the center of the rotating disk, and a slit at the center was installed to narrow further the X-ray beam. The SMS is a number of optical elements with the key element being an anti-ferromagnetic and almost ideal single crystal of iron borate $^{57}FeBO_3$, which is used in pure nuclear reflection (111), *i.e.* the reflection which is forbidden for electronic scattering but allowed for nuclear scattering. The crystal is maintained in an external magnetic field and is heated a bit below the Neel temperature, where the magnetic hyperfine structure collapses to a single-line spectrum.

2. The Mössbauer spectra of a rotating absorber

Consider a thin single-resonant Mössbauer absorber with resonant frequency ω_0. The transmission spectrum of such an absorber is given by

$$T(\omega, \omega_0) = 1 - \frac{A\gamma^2}{(\omega - \omega_0)^2 + \gamma^2}, \qquad (1)$$

where ω is the incoming frequency, γ is the half width at half absorption intensity, and A measures the relative absorption amplitude of the absorption curve. This can be expressed by use of a Lorentzian defined as $L_\gamma(\omega) = \gamma/[\pi(\omega^2+\gamma^2)]$ and then $T(\omega, \omega_0) = 1 - \pi A\gamma L_\gamma(\omega - \omega_0)$.

The SMS source beam front has a Lorentzian square shape which for mathematical simplicity will be approximated by a Lorentzian shape (this approximation will not interfere with the derived conclusions, as will be discussed later) and our absorber is also Lorentzian. Thus, the absorption is a convolution of the two Lorentzians. It is known that the convolution of two Lorentzians is a Lorentzian and $L_{\gamma_1} L_{\gamma_2} = L_{\gamma_1+\gamma_2}$. Thus, by changing the absorption line width, we can assume that our SMS source is of single frequency ω_s given by $\omega_s = \omega_0(1 + v_s/c)$, where v_s is the velocity of the source.

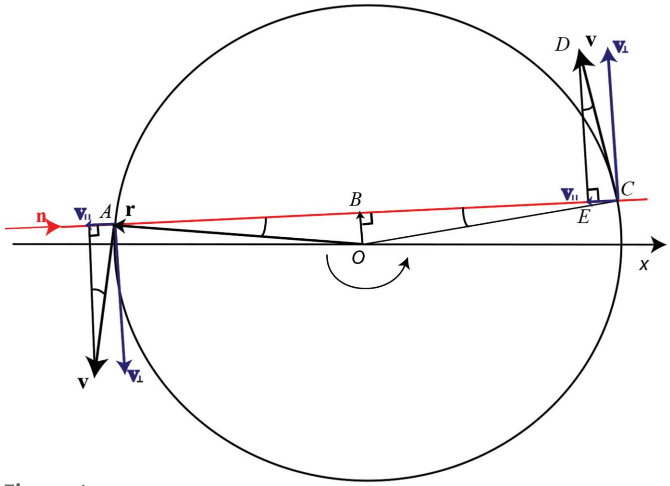


Figure 1
Velocity decomposition of a rotating absorber.

The Mössbauer absorber is placed at the rim of a fast rotating disk of radius r which is rotating with frequency ω_r. O in Fig. 1 is the rotation axis of the disk, which may differ slightly from the center of the disk. A SMS beam in the x-direction focused to O hits this rotating absorber. For any given single ray in the beam, we denote by B the point on the ray which is closest to O, and define $\mathbf{b} = \overline{OB}$. Thus, b is the distance from the rotation axis to the ray. The velocity of any point of the absorber is $\mathbf{v} = \boldsymbol{\omega}_r \times \mathbf{r}$, where $\mathbf{r}$ is the vector from the axis of rotation to the rotating point. This velocity is perpendicular to $\mathbf{r}$. We decompose this velocity into the parallel part $\mathbf{v}_\parallel$ and transverse part $\mathbf{v}_\perp$ with respect to the incoming ray (see Fig. 1). From the similarity of triangles ΔOBC and ΔCED, the parallel velocity with respect to the ray at the absorber at point A or C is given by $\mathbf{v}_\parallel = \boldsymbol{\omega}_r \times \mathbf{b}$, $v_\parallel = vb/r = \omega_r b$ and is the same for any point resting on the disk, positioned along the ray. The transverse velocity is $v_\perp = v\sqrt{r^2 - b^2}/r = \omega_r r\sqrt{1 - b^2/r^2}$.

Due to the Longitudinal and TD shifts, the resonant frequency of the rotating absorber is $\omega_a = \omega_0(1 + v_\parallel/c - v_\perp^2/2c^2)$. Denote by s_T the TD shift. Since $b \ll r$, we may assume that, in units of velocity,

$$s_T = \omega_r^2 r^2/2c. \tag{2}$$

From the decomposition of the velocity, we see that for any given ray $\omega_a(b) = (\omega_0/c)(c + \omega_r b - s_T)$ and depends only on b. This implies that $\omega_s - \omega_a = (\omega_0/c)(v_s + s_T - \omega_r b)$. By introducing $\gamma_v = c\gamma/\omega_0$, the half width in units of velocity, and

$$\tilde{v}(b) = v_s + s_T - \omega_r b, \tag{3}$$

we can rewrite the transmission spectrum (1) for the given ray as

$$T[\omega_s, \omega_a(b)] = 1 - \frac{A\gamma_v^2}{\tilde{v}(b)^2 + \gamma_v^2} = 1 - \pi A \gamma_v L_{\gamma_v}[\tilde{v}(b)].$$

We assume that the intensity distribution $I_d(b - b_0)$ of the profile of the focal spot is symmetric around b_0, the distance from the center of the beam to the rotation axis of the disk, with half width d. Then, the transmission spectrum of a sample rotating with frequency ω_r is

$$\begin{aligned} T_{\omega_r}(v_s) &= \int I_d(b - b_0)\, T[\omega_s, \omega_a(b)]\, db \\ &= 1 - \pi A \gamma_v \int L_{\gamma_v}[\tilde{v}(b)]\, I_d(b - b_0)\, db. \end{aligned}$$

Denote $\tilde{b} = \omega_r(b - b_0)$, which implies $\tilde{v}(b) = \tilde{v}(b_0) - \tilde{b}$. With these substitutions

$$\begin{aligned} T_{\omega_r}(v_s) &= 1 - \pi A \gamma_v \int L_{\gamma_v}[\tilde{v}(b_0) - \tilde{b}]\, I_{\omega_r d}(\tilde{b})\, d\tilde{b} \\ &= 1 - \pi A \gamma_v\, L_{\gamma_v}\, I_{\omega_r d}[\tilde{v}(b_0)]. \end{aligned} \tag{4}$$

Using (3) implies that $T_{\omega_r}(v_s)$ is symmetric with respect to

$$x_r = \omega_r b_0 - s_T. \tag{5}$$

If we assume now that $I_d(b - b_0)$ is approximately a normalized Lorentzian, by use of the convolution formula for Lorentzians,

$$\begin{aligned} T_{\omega_r}(v_s) &= 1 - \pi A \gamma_v\, L_{\omega_r d}\, L_{\gamma_v}[\tilde{v}(b_0)] \\ &= 1 - \pi A \gamma_v\, L_{\omega_r d + \gamma_v}[\tilde{v}(b_0)], \end{aligned}$$

which implies that the broadening and intensity parameters are

$$\gamma_r = \omega_r d + \gamma_v, \qquad A_r = A\gamma_v/\gamma_r. \tag{6}$$

The term

$$x_a = \omega_r b_0 \tag{7}$$

in (5), previously defined as the *alignment shift*, is a result of a misalignment of the system. If this shift is larger than the size of the measuring velocity v_s window, the signal is lost. On the other hand, the alignment shift helps to align the system. The deduced shift yields b_0 and directs the way to minimize b_0.

Since in (5) the total shift is the sum of the alignment shift and TD, the accurate TD value can be obtained only if one measures the spectra for both directions of rotation, namely clockwise and counterclockwise. Then, the alignment shift changes its sign whereas the TD shift is unchanged. By averaging the results obtained in both directions the accurate TD value is obtained.

3. Experimental details and results

The theoretical predictions of the effect of rotation on the Mössbauer spectra were tested in the experiments performed at beamline ID18 of the ESRF in Grenoble, France.

The rotor system was designed by a joint team of Ben-Gurion University of the Negev and the Nuclear Research Center-Negev, Beer-Sheva, Israel. The main components of the rotor system are presented in Fig. 2.

A 50 mm-radius disk was designed for rotation frequencies of up to 1 kHz. The disk is made of titanium alloy 6Al-4V and includes 108 slots, distributed extremely uniformly around the circle. The slots have been carefully cut by an electric discharge machining process, A semicircular-shape stainless steel 304 (thickness 50 μm) absorber which shows a single-line Mössbauer spectrum was placed on the rim of the disk. The

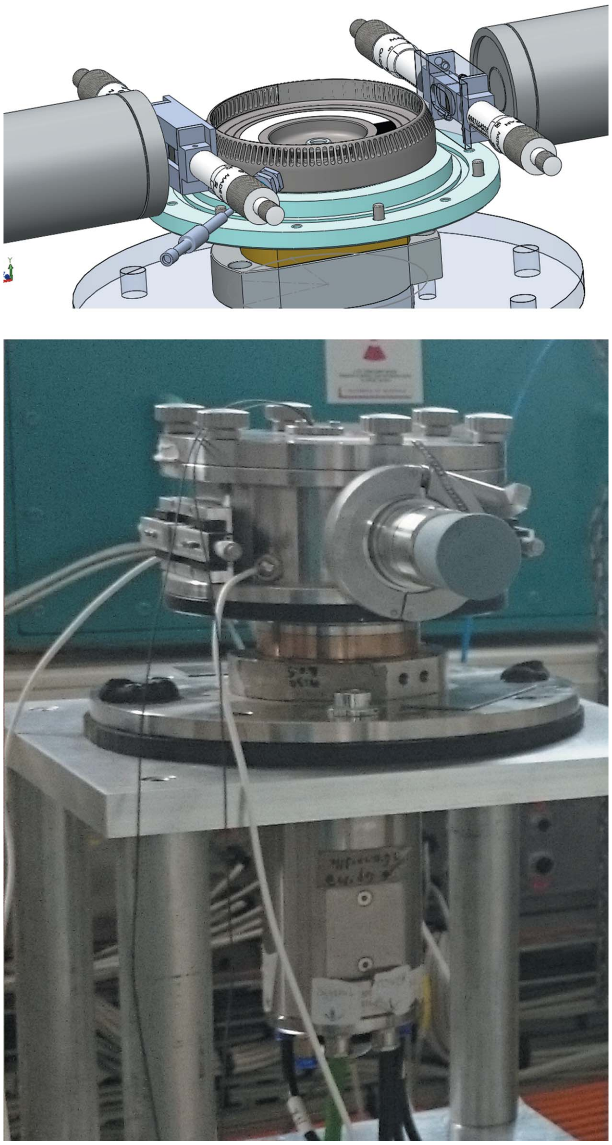


Figure 2
(Top) Sketch of the disk used in the experiment with a semicircular shape single-line stainless steel absorber, placed on the shaded part of the rim of the disk on the right-hand side (see also Fig. 5). (Bottom) Photograph of the rotor at beamline ID18.

other side of the disk rim was covered by a transparent Mylar foil for radial balancing.

An unsealed vacuum chamber was designed which allows 7 mbar internal pressure (see right-hand upper corner of Fig. 2). The disk is driven by a high-speed air-bearing spindle produced by Colibri Spindles Ltd, Israel. The spindle is located outside of the vacuum chamber while the driving shaft crosses the chamber wall through a carefully designed bore, leaving a gap of 50 µm between the static (chamber) and dynamic (shaft) parts. Vacuum conditions, essential for high-speed rotation, low friction and acoustic noise, are maintained by an automatically controlled vacuum pump. The final radial balancing, performed by Colibri, was achieved by material removing from dedicated balancing areas at the disk surface, while the disk was assembled on the spindle shaft.

The rotor system was specially designed to preserve constant rotation frequencies, clockwise and counter-clockwise, and to provide information on the absorber orientation. Two proximity sensors (by Micro-Epsilon Ltd) were placed orthogonal to one another and measure the radial displacements of the disk. An optical sensor mounted on top of the chamber and facing the disk allows dividing each rotation into two rotating stages that reflect the circular position of the rotating absorber.

The rotating system was incorporated in beamline ID18 of ESRF. The energy position and energy width of the SMS were stable during measurements with the accuracy of their determination. Between each set of measurements we obtained the spectrum of a reference single-line absorber to ensure the stability of the energy position and the energy width for the SMS. The changes were small and corrected, when needed. First, we tested the broadening formulas (6) using the beam of the SMS after passing the KB optics, which was used to focus the beam on the center of the disk. We found that the line broadening fits the predicted formulas, with $2d$ = 15.6 µm. Focusing on the rotation axis was accomplished by use of (7) and observing the alignment shift x_a. From the observed shift at a given step, we calculated b_0, the distance from the beam to the rotation axis, and corrected the rotor position to align the beam to the rotation axis at the next step. In order to measure a statistically meaningful absorption spectrum, our rotation frequency was limited to 100 Hz.

To obtain information at higher frequencies, a slit made of Re metal was mounted at the center of the disk. The use of this slit reduced the beam broadening to $2d$ = 5.4 µm and enabled us to obtain statistically significant spectra for rotation frequencies up to 300 Hz. Fig. 3 shows the Mössbauer transmission lines measured by conventional MS and by SMS obtained at rest, and at rotating frequencies of 22 Hz and 200 Hz measured with the slit. One can observe a noticeable (-0.9 mm s^{-1}) shift between the center of the spectrum at 200 Hz relative to that at rest. From (2) it follows that the TD shift in this case is s_T = 0.007 mm s^{-1}. Hence, the shift is mainly the alignment shift, equation (7), from a misalignment of b_0 = 0.7 µm.

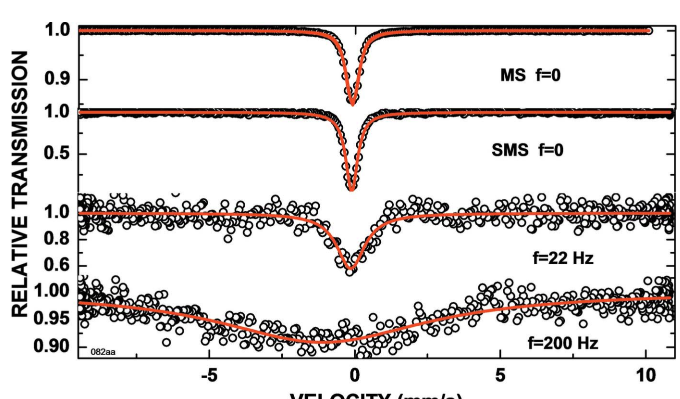


Figure 3
Mössbauer transmission spectra measured in relative transmission by conventional MS at rest and by SMS at rest and at 22 Hz and 200 Hz.

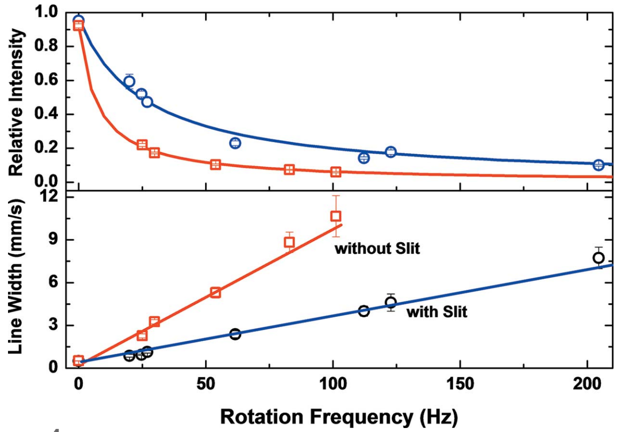


Figure 4
The total width $2\gamma_r$ and relative absorption amplitude A_r of a rotating absorber focused with KB optics, with and without a slit.

To verify the broadening of the absorption line formula (6), absorption lines at various frequencies with and without the slit have been measured. Fig. 4 exhibits the experimental width at half amplitude $2\gamma_r$ and the relative absorption amplitude A_r as a function of the frequency.

4. Method of testing transverse Doppler shift and clock hypothesis with a rotating absorber

In addition to velocity, causing a TD shift, a rotating absorber also experiences a significant acceleration. Thus, the rotating absorber experiments could be used to test Einstein's clock hypothesis (CH). It was shown by Friedman (2011) that if the CH is not true then a Doppler type shift due to acceleration will be observed. Such a shift could be observed by measuring the shift between the absorption spectra of two sides of a rotating Mössbauer absorber.

To do this, a semicircular Mössbauer absorber A should be placed on the rim of a disk. The detector D will be diametrically opposed to the SMS source, as shown in Fig. 5. The beam should be focused on the center of the disk by use of KB optics and a slit, positioned at the center of the disk. The disk with the absorber will be rotated with a high-speed vibrationless spindle with several angular velocities ω_r. We need to separate the counts when the acceleration of the absorber is opposing the direction of the radiation, as in case (*a*) of Fig. 5, from the counts when the acceleration of the absorber is in the direction of the radiation, as in case (*b*) of Fig. 5. Two separate absorption curves will be obtained for each case. Such a separation of signals has been performed in our present experiment, but the stability and accuracy (statistics) was not enough to make definite conclusions.

The radiation from the SMS source undergoes three shifts: the known TD shift, the new alignment shift and an expected shift due to the absorber acceleration. The expected third shift is

$$s_a = r\omega_r^2/a_m, \tag{8}$$

where a_m is the unknown maximal acceleration. The first two shifts are the same for both cases, as can be seen from Fig. 1, while the third shift changes sign from case (*a*) to (*b*). If the CH is valid, no shift between the two absorption curves should be observed. On the other hand, if such a shift beyond the uncertainties of the experimental values is observed, that will serve as direct proof that the acceleration influences the observed rate of the clock and that the CH is not valid. Moreover, this experiment will enable the value of the maximal acceleration to be determined.

It is necessary to perform a set of experiments by gradually increasing the rotational frequency. At each rotation frequency, the shift observed will direct us to calculate b_0 [by (2)] and to align the rotation axis to coincide with the beam center. Then, by applying (6) one should check whether the current slit size is good enough to obtain a statistically meaningful spectrum. If not, narrowing the slit will be needed.

To be able to measure accurately the TD shift, two experiments at a sufficiently large rotation frequency in opposite directions are requested. From (5) it follows that the average shift in both experiments will be equal to the TD shift.

5. Discussion and conclusions

In §2 we have shown that the absorption lines of a rotating single-line absorber are broadened with respect to the rest line, and there is an additional shift due to the alignment of the system. Since the broadening depends only on the longitudinal Doppler shift, which vanishes for rays passing through the rotation axis of the disk, focusing the X-ray beam to the rotation axis could significantly reduce this broadening. This can be achieved by using (i) KB optics focused on the center of the disk and (ii) a slit positioned at the center of the disk blocking rays distant from the center. Obviously, a narrow slit will reduce the counts of the experiment, implying longer testing time.

The transmission spectrum of a sample rotating with frequency ω_r is given by formula (4) for arbitrary intensity distribution of the profile of the focal spot. Assuming the distribu-

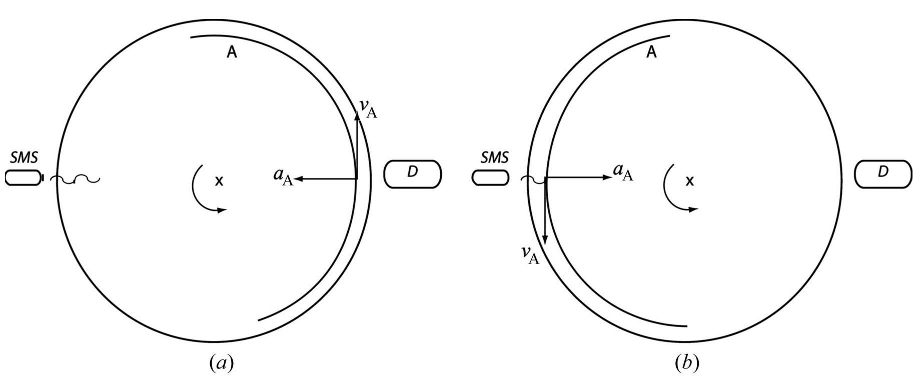


Figure 5
The semicircular absorber is labeled (A). The acceleration in (*a*) is anti-parallel, and in (*b*) is parallel, to the SMS radiation.

tion is symmetric, the isomer shift of this transmission spectrum is given by formula (5). This formula is the one needed to test the time dilation. Assuming the distribution is Lorentzian, the broadening is expressed by equation (6). Fig. 4 shows that the value of A_r, the relative absorption amplitude during rotation, which depends on the angular velocity ω_r and on the width d of the beam at the center of the disk, decreases very fast with the increase of ω_r. Also, the absorption line width $2\gamma_r$ increases with ω_r. This implies that for large ω_r the transmission line T_{ω_r} becomes unobservable. Thus, for statistically meaningful measurements, the rotational frequency for any given beam width is limited.

The additional alignment shift is given by (7). This shift becomes significantly large with the increase of the rotational frequency and can lead to the disappearance of the spectrum line in the velocity v_s observation window. Nevertheless, we can use this shift to find the displacement of the rotation axis with respect to the center of the beam and to improve the alignment. In §3 we presented the experimental verification of the formulas of broadening and the alignment shift. From the discussion of Fig. 3, one sees that at 200 Hz a misalignment of less than 1 μm can be easily detected and corrected.

In §4 we propose a method of testing the transverse Doppler shift and the clock hypothesis with a rotating Mössbauer absorber. Since the shifts are proportional to the disk radius and the rotational frequencies are limited by the broadening, we cannot use disks with small radii.

For testing of the CH, with a rotating disk of radius 50 mm at 1 kHz, and using our estimate (Friedman & Gofman, 2010) $a_m = 10^{19}$ m s^{-2}, from (8), the shift between the two spectra should be

$$2s_a = 2\frac{\omega_r^2 r}{a_m} = \frac{2(2\pi \times 10^3)^2 \times 5 \times 10^{-2}}{10^{19}} \approx 4 \times 10^{-13},$$

which corresponds to a shift of $s = 2s_a c \approx 0.12$ mm s^{-1}. This also explains why, in our present experiments, measured up to 300 Hz, in which we observed the predicted (Friedman & Nowik, 2012) broadening and the predicted new alignment shift, we were not able to make any conclusions concerning the TD and acceleration shift. To be able to measure a significant shift, the rotation frequency has to be close to 1 kHz and the slit has to be of about 1–2 μm width. Only use of SMS (and not conventional MS) can provide a significant counting rate with such a small slit. We intend to continue this project with better conditions, an enriched absorber and a small adjustable slit.

If the non-zero value for the maximal acceleration is experimentally confirmed, this result would have a major impact on the relativity theory and its implications. For possible applications, see Friedman (2013).

Acknowledgements

We wish to acknowledge the financial support of the Jerusalem College of Technology research fund and ESRF. We would like to thank T. Scarr for his proofs. YF and EY were supported in part by the German–Israel Foundation for Scientific Research and Development: GIF No. 1078-107.14/2009.

References

Champeney, D. C., Isaak, G. R. & Khan, A. M. (1965). *Proc. Phys. Soc.* **85**, 583–593.
Champeney, D. C. & Moon, P. B. (1961). *Proc. Phys. Soc.* **77**, 350–352.
Cranshaw, T. E. & Hay, H. J. (1963). *Proceedings of the International School of Physics 'Enrico Fermi'*, p. 220. New York: Academic Press.
Cranshaw, T. E. Schiffer, J. P. & Egelstaff, P. A. (1960). *Phys. Rev. Lett.* **4**, 163–164.
Einstein, A. (1911). *Ann. Phys.* **35**, 898–908.
Friedman, Y. (2011). *Ann. Phys. (Berlin)*, **523**, 408–416.
Friedman, Y. (2013). *J. Phys. Conf. Ser.* **437**, 012017.
Friedman, Y. & Gofman, Yu. (2010). *Phys. Scr.* **82**, 015004.
Friedman, Y. & Nowik, I. (2012). *Phys. Scr.* **85**, 065702.
Hay, H. J. (1962). *Proceedings of the 2nd Conference on the Mössbauer Effect*, edited by A. Schoen and D. M. T. Compton, p. 225. New York: Wiley.
Hay, H. J. *et al.* (1960). *Phys. Rev. Lett.* **4**, 165–166.
Kholmetskii, A. L., Yarman, T., Missevitch, O. V. & Rogozev, B. I. (2009). *Phys. Scr.* **79**, 065007.
Kirkpatrick, P. & Baez, A. V. (1948). *J. Opt. Soc. Am.* **38**, 766–774.
Kündig, W. (1963). *Phys. Rev.* **129**, 2371.
Potapkin, V., Chumakov, A. I., Smirnov, G. V., Celse, J.-P., Rüffer, R., McCammon, C. & Dubrovinsky, L. (2012). *J. Synchrotron Rad.* **19**, 559–569.